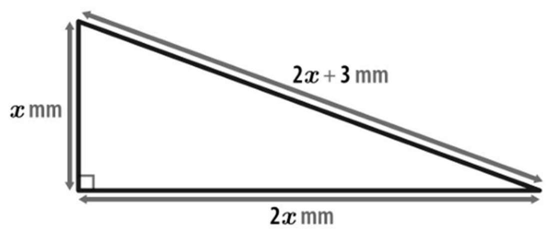
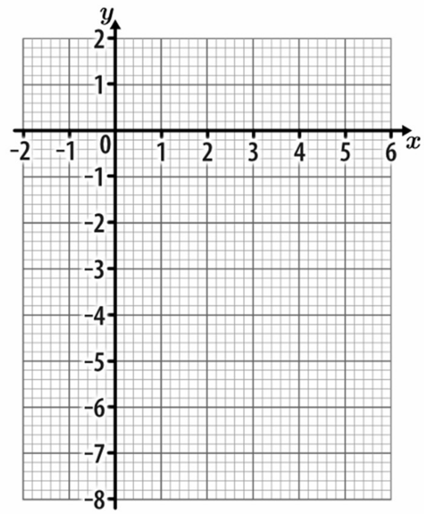
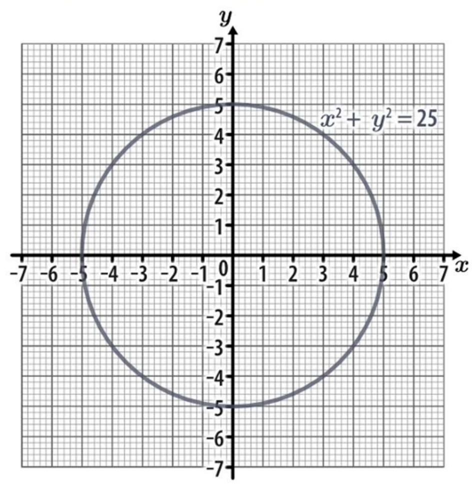
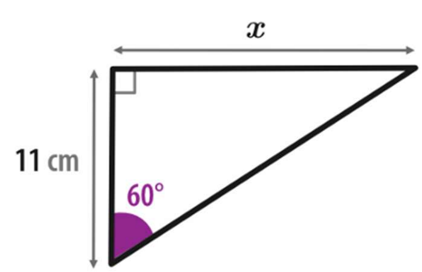
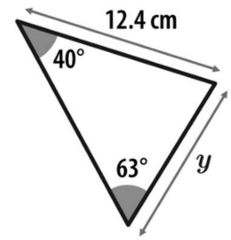
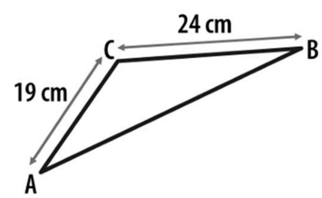
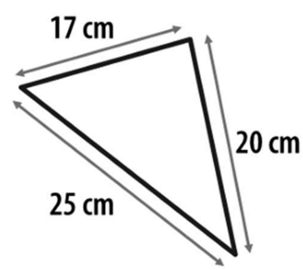
Maths Knowledge Organiser

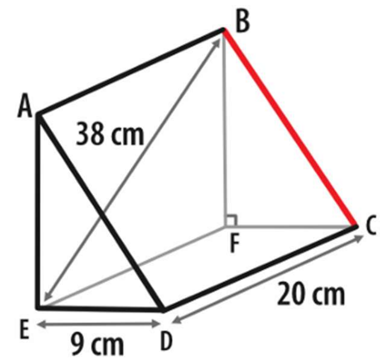
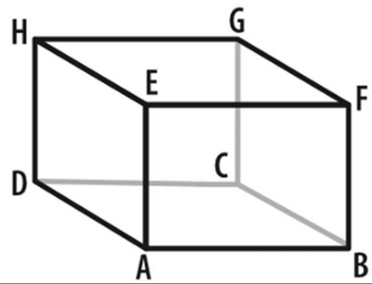
Year: 11 Term: Autumn (1 and 2)

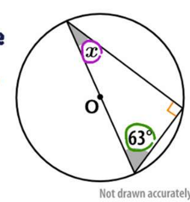
Unit	Topic/Skill	Definition/Tips	Example	Sparx Code
Equations	Factorising to solve quadratic equations of the form $ax^2 + bx + c = 0$	Split the x term into two, by finding a pair of numbers that add to b, and multiply to give ac. Start with the factor pairs of ac.	Solve $2x^2 + 16x + 30 = 0$	U960
	Solving quadratic equations by completing the square	Write $ax^2 + bx + c = 0$ in the form $p(x + q)^2 + r = 0$	Solve $-2x^2 = 12x + 5$ Give your answers fully simplified in the form $x = \frac{m+\sqrt{n}}{p}$, where m, n and p are integers.	U589
	Solving quadratic equations using the quadratic formula	Be careful when b is negative. e.g. If $b = -4$, then $-b = 4$, and $b^2 = 16$.	Solve $2x^2 - 17x + 14 = 0$ Give your answers to 1 decimal place.	U665
	Constructing and solving quadratic equations	Use brackets when you mean to square an expression with two terms. e.g. $(2x + 3)^2$	A right-angled triangle is shown below. Calculate the length of the shortest side. Give your answer to 2 decimal places. 	U150
	Solving quadratic equations graphically	Use brackets or the table function when completing a table of values *using a calculator.*	Plot the graph of $y = x^2 - 4x - 4$ Use your graph to estimate the solutions to the equation $x^2 - 4x - 4 = 0$ Give your answers to 1 decimal place.	U601

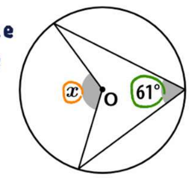
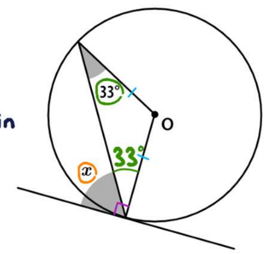
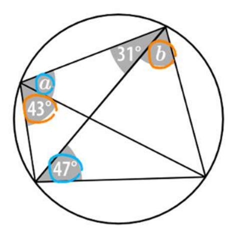
				
	Solving simultaneous equations involving quadratics	Substitute the linear equation into the quadratic equation.	Solve the following simultaneous equations: $y = 2x + 3$ $x^2 + 4y - 45 = 0$	U547
	Solving simultaneous equations involving quadratics graphically	The intersections between two graphs represent the solutions when their equations are solved simultaneously.	The diagram below shows the graph of $x^2 + y^2 = 25$ Use the diagram to estimate the solutions to these simultaneous equations: $x^2 + y^2 = 25$ $y = 2x + 1$ Give your answers to 1 decimal place. 	U875

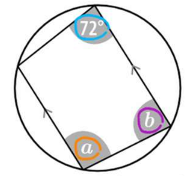
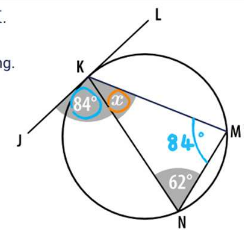
Unit	Topic/Skill	Definition/Tips	Example	Sparx Code
Trigonometric Ratios & Graphs	Using the exact values of trigonometric ratios - Higher	You need to learn the exact values of sin cos and tan of $0°, 30°, 45°, 60°$ & .	Work out the length x 	U319

Trigonometric Ratios & Graphs	Using trigonometric graphs	The period of sin and cos is 360°, but the period of tan is 180°.	Sketch the graph of $y = \sin x$ for $0^\circ \leq x \leq 360^\circ$ Use your graph to solve the following for $0^\circ \leq x \leq 360^\circ$ a) $\sin x = 0$ b) $\sin x = -1$	U450
Non-right-angled Trigonometry	The sine rule	The formula appears on the formula sheet like this: sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ You usually only need to relate two pairs of sides and opposite angles. Flip the fractions when you are finding angles.	Use the sine rule to calculate the length y. Give your answer to 1 d.p. 	U952
Non-right-angled Trigonometry	The cosine rule	The formula appears on the formula sheet like this: cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$ Side a is opposite angle A.	Triangle ABC has an area of $160\ \mathrm{cm}^2$. Angle ACB is obtuse. Calculate the size of angle ACB to 1 d.p. 	U591
Non-right-angled Trigonometry	The area rule	The formula appears on the formula sheet like this: Area of triangle = $\frac{1}{2} a b \sin C$ Angle C is in between sides a and b.	Calculate the area of the triangle to 1 d.p. 	U592

3D Pythagoras' Theorem & Trigonometry	Using Pythagoras' theorem in 3D	Draw the 2D right-angled triangle that you are using at every step.	Calculate the length BC in the triangular prism. Give your answer to 1 d.p.  	U541
3D Pythagoras' Theorem & Trigonometry	Trigonometry in 3D shapes	Draw the 2D right-angled triangle that you are using at every step.	In the cuboid below, $\mathrm{DF} = 51\,\mathrm{cm}$ $\mathrm{AF} = 37\,\mathrm{cm}$ Work out the size of angle AFD to 1 d.p.  	U170

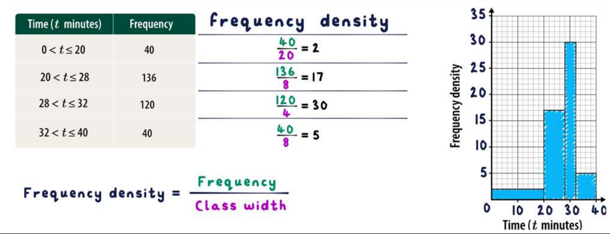
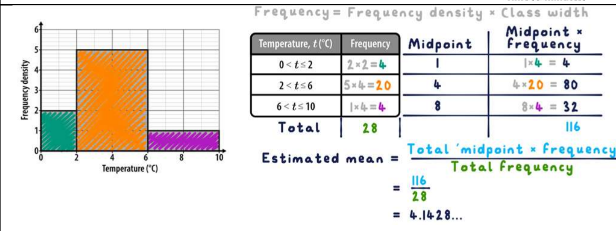
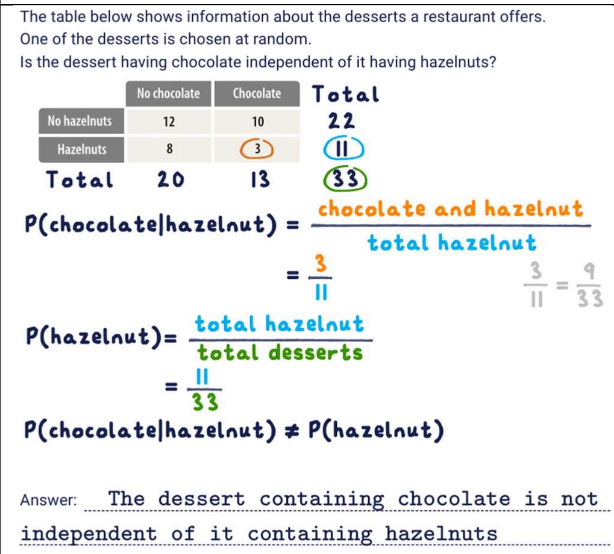
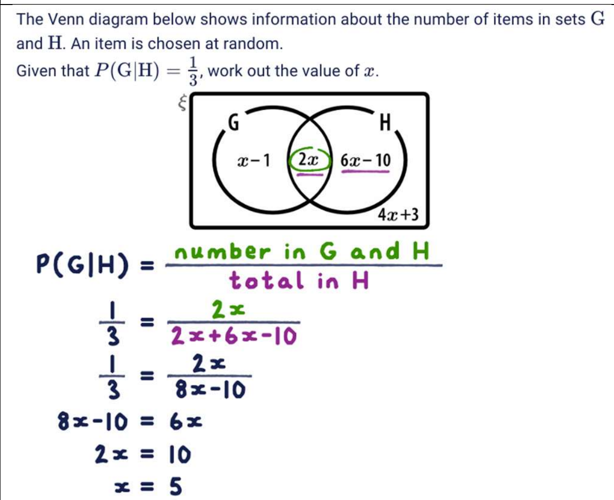
Unit	Topic/Skill	Definition/Tips	Example	Sparx Code
Circle Theorems	Finding angles at the circumference	The angle at the circumference of a semicircle is a right angle	Work out the size of angle x. Give a reason for each stage of your working. the angle at the circumference in a semicircle is a right angle and angles in a triangle add to 180° so $x + 90 + 63 = 180$ $x = 27$  	U459

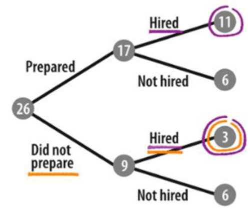
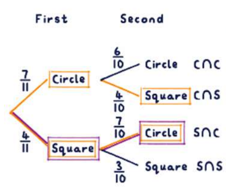
Circle Theorems	Finding angles subtended at the centre or circumference of a circle	The angle at the centre is twice the angle at the circumference when subtended by the same arc.	Work out the size of angle x. Give a reason for your answer.  Not drawn accurately the angle at the centre is twice the angle at the circumference so $x = 2 \times 61$ $x = 122$ Answer: 122 °	U459
Circle Theorems	The radius meeting a tangent forms a right angle	Triangles inside a circle are isosceles if two of the sides are radii	Work out the size of angle x. Give a reason for each stage of your working.  Not drawn accurately isosceles triangles contain two equal sides and two equal angles and the angle between a tangent and radius is 90° so $x + 33 = 90$ $x = 57$ Answer: 57 °	U489
Circle Theorems	Angles in the same segment are equal	The angle must be on the circumference and subtended by the same arc	Work out the sizes of angles a and b. Give a reason for your answers.  Not drawn accurately angles in the same segment are equal so $a = 47$ and $b = 43$ Answer: Angle a: 47 ° Angle b: 43 °	U251

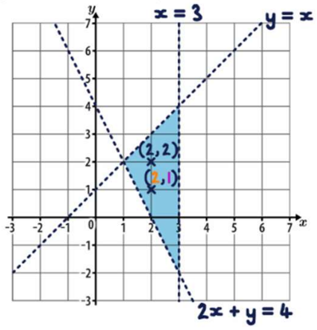
Circle Theorems	The opposite angles in a cyclic quadrilateral sum to 180°	The four vertices of the cyclic quadrilateral **<u>must</u>** be on the circumference	Work out the sizes of angles a and b. Give reasons for your answers.  Not drawn accurately opposite angles in a cyclic quadrilateral add to 180° so $a + 72 = 180$ $a = 108$ co-interior angles add to 180° so $a + b = 180$ $108 + b = 180$ $b = 72$ Answer: Angle a: 108 ° Angle b: 72 °	U251
Circle Theorems	The alternate segment theorem	The angle that lies between a tangent and a chord is equal to the angle subtended by the same chord in the alternate segment.	Line JL is a tangent to the circle at point K. Work out the size of angle x. Give a reason for each stage of your working.  Not drawn accurately using the alternate segment theorem Angle KMN = 84 and angles in a triangle add to 180° so $x + 62 + 84 = 180$ $x = 34$ Answer: 34 °	U130

Maths Knowledge Organiser

Year: 11 Term: Spring (3 and 4)

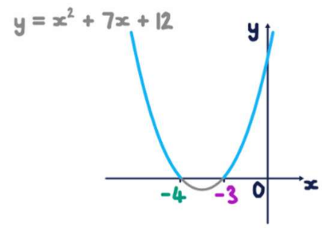
<table>
<tr><th>Unit</th><th>Topic/Skill</th><th>Definition/Tips</th><th>Example</th><th>Sparx Code</th></tr>
<tr><td>Statistical Diagrams</td><td>Drawing and interpreting histograms with equal and unequal class widths</td><td>Frequency density= frequency ÷ class width</td><td>
</td><td>U185
U814</td></tr>
<tr><td></td><td>Finding averages from histograms</td><td>Draw a table if there isn’t already one and then calculate the mean using mp x f</td><td>
</td><td>U983
U267</td></tr>
<tr><td>Probability</td><td>Conditional probabilities from tables</td><td>P(A|B) is the probability of A given B.
If events A and B are independent, then P(A|B) = P(A) .</td><td>
</td><td>U246</td></tr>
<tr><td></td><td>Conditional probabilities from Venn diagrams</td><td>A Venn diagram helps you to visualise conditional probabilities.</td><td>
</td><td>U699</td></tr>
</table>

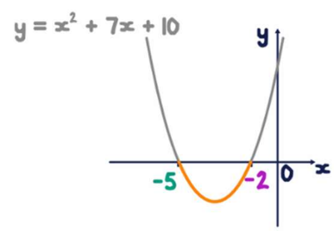
	Using the conditional probability formula	$P(A\|B) = \frac{P(A \cap B)}{P(B)}$	$P(\mathrm{A}\|\mathrm{B}) = \frac{3}{5} \qquad P(\mathrm{B}) = \frac{4}{7}$ Calculate $P(\mathrm{A} \cap \mathrm{B})$ $P(A\|B) = \frac{P(A \cap B)}{P(B)}$ $\frac{3}{5} = P(A \cap B) \div \frac{4}{7}$ $P(A \cap B) = \frac{3}{5} \times \frac{4}{7}$ $= \frac{12}{35}$	U821
	Conditional probabilities from tree diagrams	Frequency tree and probability tree diagrams can both be used to solve problems involving conditional probability.	The frequency tree below shows some information about people who applied for a job. A person is picked at random. Given that the person was hired, work out the probability that they did not prepare.  $P(\text{not prepared}\|\text{hired}) = \frac{\text{number not prepared and hired}}{\text{total hired}}$ $= \frac{3}{11+3}$ $= \frac{3}{14}$  $P(S \cap C) = \frac{4}{11} \times \frac{7}{10}$ $= \frac{28}{110}$ $P(\text{not same}) = P(C \cap S) + P(S \cap C)$ $= \frac{7}{11} \times \frac{4}{10} + \frac{28}{110}$ $= \frac{28}{110} + \frac{28}{110}$ $= \frac{56}{110}$ $P(S \cap C\|\text{not same}) = \frac{P(S \cap C)}{P(\text{not same})}$ $= \frac{28}{110} \div \frac{56}{110}$ $= \frac{28}{\cancel{110}} \times \frac{\cancel{110}}{56}$ $= \frac{28}{56}$ Answer: $\frac{28}{56}$ or $\frac{1}{2}$	U806
	Using the product rule for counting	**Permutations** require a specific sequence or order, while **combinations** are just selections where order does not matter.	Using the digits 0 to 6 inclusive, work out how many different 4-digit numbers it is possible to make if each digit can appear only once. 1^{st} 2^{nd} 3^{rd} 4^{th} $= 7 \times 6 \times 5 \times 4$ $= 840$ A clothing store sells 6 types of shirt and several types of trousers. There are 84 possible combinations of a shirt and trousers. How many types of trousers does the store sell? types of shirt × types of trousers = 84 $6 \times t = 84$ $t = 84 \div 6$ $t = 14$	U369

<table>
<tr>
<td>Inequalities</td>
<td>Graphs of linear inequalities</td>
<td>For < or > use a dashed line.
For ≤ or ≥ use a solid line.
Shade the region that you do want.</td>
<td>
a) On the grid, shade the region which satisfies all these inequalities
$y < x + 1$ $2x + y > 4$ $x < 3$


y = x + 1
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>y</td><td>1</td><td>2</td><td>3</td></tr>
</table>
2x + y = 4
<table>
<tr><td>x</td><td>0</td><td>2</td></tr>
<tr><td>y</td><td>4</td><td>0</td></tr>
</table>
Check (2,1) satisfies the inequalities
$y < x + 1$
$1 < 2 + 1$
$1 < 3$ ✓
$2x + y > 4$
$2 \times 2 + 1 > 4$
$5 > 4$ ✓
$x < 3$
$2 < 3$ ✓
b) List all of the points with integer coordinates which satisfy the three inequalities.
Answer: (2, 1) and (2, 2)
</td>
<td>U747</td>
</tr>
<tr>
<td></td>
<td>Solving quadratic inequalities</td>
<td>Solve the ‘equation’ to find the x intercepts and sketch the curve.

For <0 find the x coordinates corresponding to the negative y coordinates

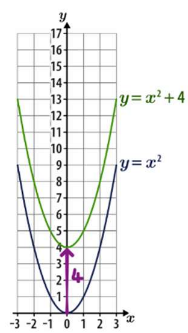
For >0 find the x coordinates corresponding to the positive y coordinates</td>
<td>
Solve the inequality $x^2 + 7x + 12 > 0$
$x^2 + 7x + 12 = 0$
$(x + 3)(x + 4) = 0$
$x + 3 = 0$ or $x + 4 = 0$
$x = -3$ or $x = -4$
$3 \times 4 = 12$
$3 + 4 = 7$


$x^2 + 7x + 12 > 0$
$y > 0$
$x < -4$ or $x > -3$
Answer: $x < -4$ or $x > -3$
Solve the inequality $x^2 + 7x + 10 \leq 0$
$x^2 + 7x + 10 = 0$
$(x + 2)(x + 5) = 0$
$x + 2 = 0$ or $x + 5 = 0$
$x = -2$ or $x = -5$
$2 \times 5 = 10$
$2 + 5 = 7$


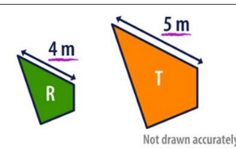
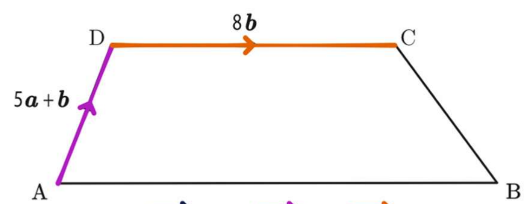
$x^2 + 7x + 10 \leq 0$
$y \leq 0$
$-5 \leq x \leq -2$
Answer: $-5 \leq x \leq -2$
</td>
<td>U133</td>
</tr>
</table>

Functions	Substituting into functions	x is the input and $h(x)$ is the output.	The function h is shown below. $h(x) = 3x^2 + 5$ Work out the value of $h(-2)$ $h(x) = 3x^2 + 5$ $h(-2) = 3\times(-2)^2 + 5$ $= 3\times 4 + 5$ $= 12 + 5$ $= 17$ $g(x) = 4x + 11$ Work out the value of x for which $g(x) = 39$ $g(x) = 4x + 11$ $39 = 4x + 11$ $-11 \mid 28 = 4x \mid -11$ $\div 7 \mid 7 = x \mid \div 7$	U637
	Substituting into composite functions	fg(2) means substitute into g first and then f gf(2) means substitute into f first and then g	The functions $f(x)$ and $g(x)$ are shown below. $f(x) = x^2 + 8 \qquad g(x) = 5x - 7$ Calculate the value of a) $fg(2)$ $fg(2) = f(g(2))$ $g(x) = 5x - 7$ $g(2) = 5\times 2 - 7$ $= 3$ $f(x) = x^2 + 8$ $f(g(2)) = g(2)^2 + 8$ $f(3) = 3^2 + 8$ $= 17$ Answer: 17 b) $gf(4)$ $gf(4) = g(f(4))$ $f(x) = x^2 + 8$ $f(4) = 4^2 + 8$ $= 24$ $g(x) = 5x - 7$ $g(f(4)) = 5\times f(4) - 7$ $g(24) = 5\times 24 - 7$ $= 113$ Answer: 113	U895
	Finding functions	You can substitute functions into other functions to make composite functions	The functions f and g are shown below. $f(x) = \sqrt{x}$ $g(x) = 2x + 7$ a) Work out $fg(x)$ $fg(x) = f(g(x))$ $g(x) = 2x + 7$ $f(x) = \sqrt{x}$ $f(g(x)) = \sqrt{g(x)}$ $f(2x+7) = \sqrt{2x+7}$ Answer: $fg(x) = \sqrt{2x+7}$ b) Work out $gf(x)$ $gf(x) = g(f(x))$ $f(x) = \sqrt{x}$ $g(x) = 2x + 7$ $g(f(x)) = 2\times f(x) + 7$ $g(\sqrt{x}) = 2\times\sqrt{x} + 7$ $= 2\sqrt{x} + 7$ Answer: $gf(x) = 2\sqrt{x} + 7$	U448

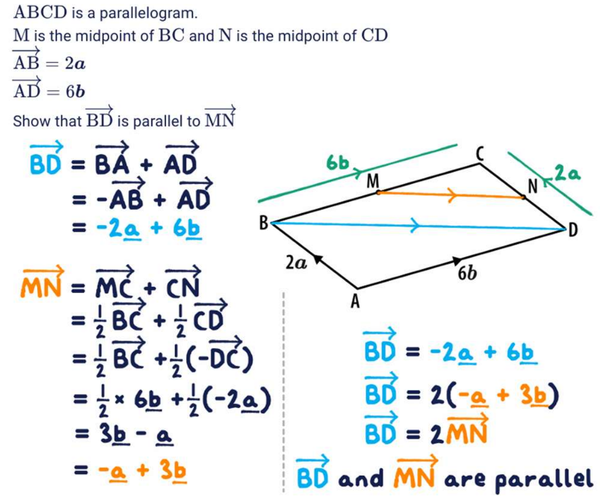
	Finding inverse functions	For complex functions make x the subject to find the inverse function. Remember to replace x with $g^{-1}(x)$ and y with x at the end. $g^{-1}(x)$ is the inverse of $g(x)$	Given that $g(x) = \sqrt{\frac{x-6}{7}}$, find an expression for $g^{-1}(x)$ $g(x) = \sqrt{\frac{x-6}{7}}$ $y = \sqrt{\frac{x-6}{7}}$ $\square^2$ $\times 7$ $+6$ $y^2 = \frac{x-6}{7}$ $7y^2 = x - 6$ $7y^2 + 6 = x$ $x = 7y^2 + 6$ $g^{-1}(y) = 7y^2 + 6$ so $g^{-1}(x) = 7x^2 + 6$ Answer: $g^{-1}(x) =$ $7x^2 + 6$	U996

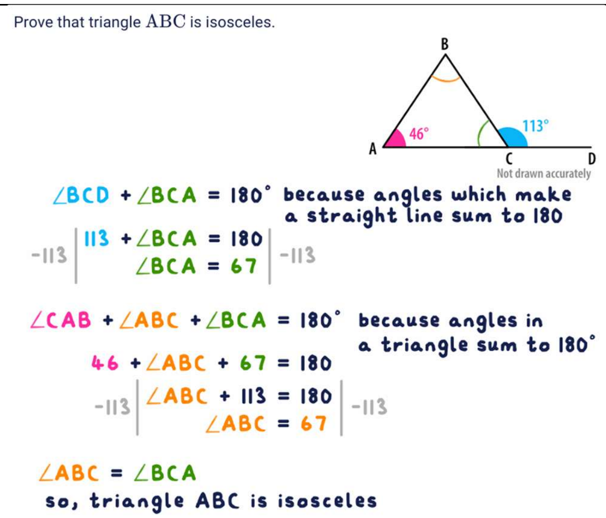
Unit	Topic/Skill	Definition/Tips	Example	Sparx Code
Transforming graphs	Translating graphs Reflecting graphs Transforming graphs	f(x) + c moves a graph up by c f(x + d) moves a graph left by d. f(x - e) moves a graph right by e.	Fully describe the transformation from $y = x^2$ to $y = x^2 + 4$  Answer: A Translation of $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$	U598 U487 U455
Iterative process	Using recurrence relations Substituting into iterative formulae Finding approximate solutions to equations using iteration	Put the starting value into the calculator and press =. Then press the ANS button where it says Xn and put the equation into the calculator. Press = for the next value(s)	The rule for a sequence is $x_{n+1} = x_n^2 + 4x_n - 3$ If $x_1 = 2$, work out the values of x_2 and x_3. $n = 1,$ $x_2 = x_1^2 + 4x_1 - 3$ $x_2 = 2^2 + 4 \times 2 - 3$ $x_2 = 9$ $n = 2,$ $x_3 = x_2^2 + 4x_2 - 3$ $x_3 = 9^2 + 4 \times 9 - 3$ $x_3 = 114$	U171 U434 U168

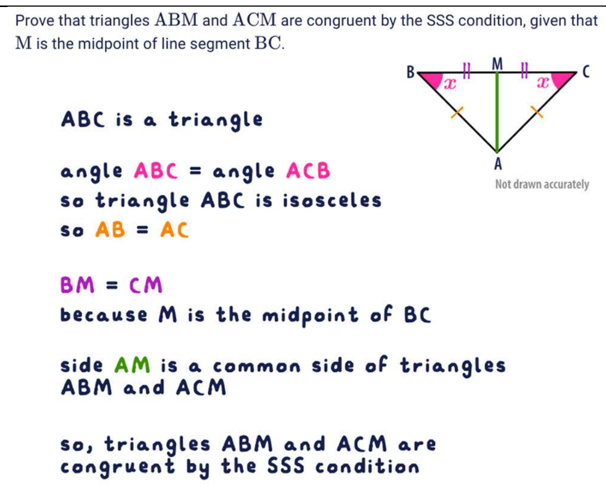
Algebraic proof	Writing algebraic proof	For identities, manipulate one side until it looks like the other. For even numbers, get the equation as 2n. For odd numbers, get the equation as 2n+1.	Prove that, for all values of n $(n+3)^2 - 6n - 5 \equiv n^2 + 4$	U582

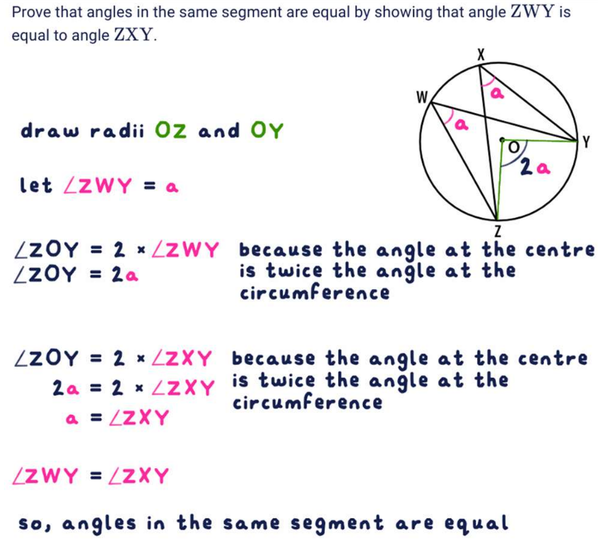
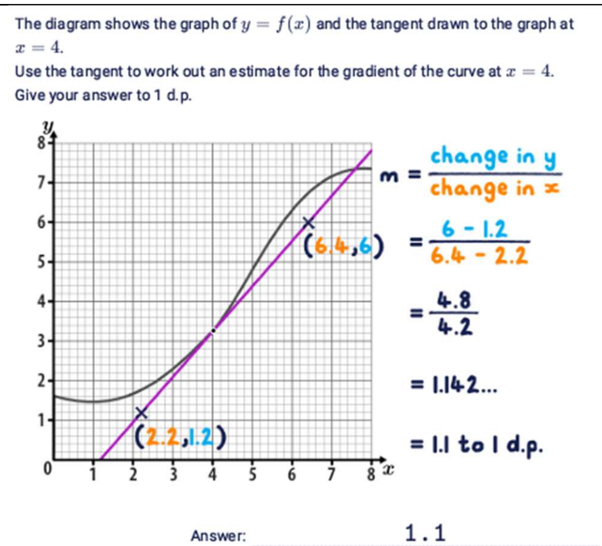
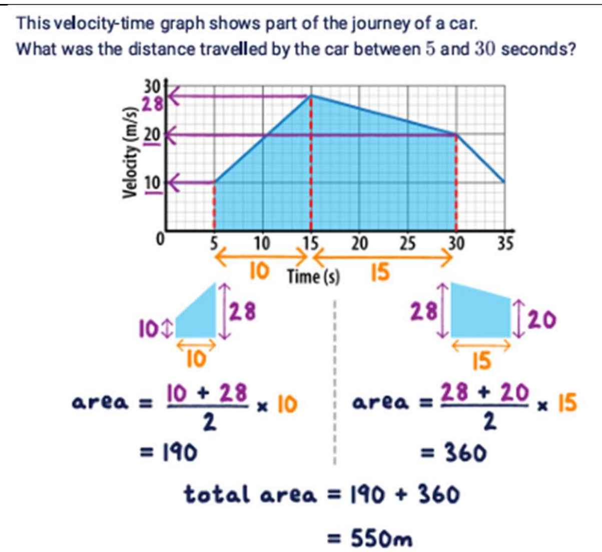
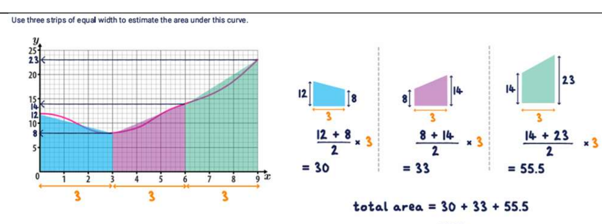
Unit	Topic/Skill	Definition/Tips	Example	Sparx Code
Similarity	Area and volume of similar shapes	The area scale factor is the square of the length scale factor. The volume scale factor is the cube of the length scale factor. The ratio of the lengths of the shapes can be squared or cubed to find the ratio of the areas or volumes respectively.	Shapes R and T are similar. Shape R has an area of $24\ \mathrm{m}^2$ Calculate the area of shape T  4 × scale factor = 5 scale factor = $\frac{5}{4}$ Area of R = 24 Area of T = Area of R × $\left(\frac{5}{4}\right)^2$ = 24 × $\frac{25}{16}$ = 37.5	U630 U110
Vector proofs	Solving geometric problems using vectors	The vectors between two vertices can be found by identifying a path from one to the other then adding the vectors along this path. For questions involving ratios, consider the fraction of the	ABCD is a trapezium. $\overrightarrow{AD} = 5\boldsymbol{a} + \boldsymbol{b}$ $\overrightarrow{DC} = 8\boldsymbol{b}$ Express $\overrightarrow{AC}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$. Give your answer in its simplest form.  $\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$ $= 5\underline{a} + \underline{b} + 8\underline{b}$ $= 5\underline{a} + 9\underline{b}$	U781

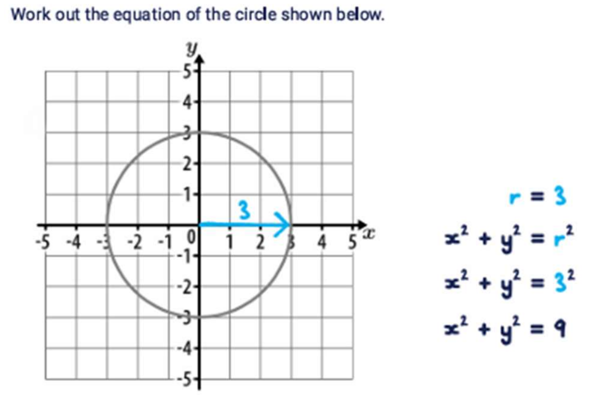
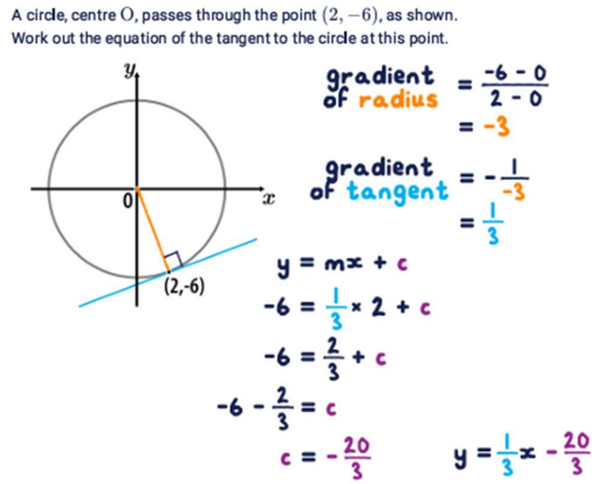
<table>
<tr><td></td><td></td><td>vector that is required.</td><td></td><td></td></tr>
<tr><td>Vector proofs</td><td>Geometric proofs with vectors</td><td>Vectors are parallel if one is a multiple of the other.

Be careful about whether vectors are positive or negative</td><td>
</td><td>U560</td></tr>
<tr><td>Writing geometric proofs</td><td>Geometric proofs with angle facts</td><td>Angles can be defined using three letters.

Ensure all reasons are written clearly using keywords – the exam board requires this to include the word “angles”</td><td>
</td><td>U471</td></tr>
<tr><td>Writing geometric proofs</td><td>Geometric proofs with congruence and similarity</td><td>Triangles can be proved to be congruent using ASA, SAS, or SSS. If they have been proved congruent then all other angles and sides must also be equal.

Triangles with three angles the same are similar, meaning their sides are in a consistent ratio.</td><td>
</td><td>U887</td></tr>
</table>

Writing geometric proofs	Proving circle theorems	Ensure that every step is clear and all reasons are properly written		U807
Non-linear graphs	Estimating gradients of non-linear graphs using tangents	Gradient of a non-linear graph can be found by drawing a line at the relevant point, choosing points that are further apart generally makes for a more accurate result.		U800
Non-linear graphs	Calculating distances from velocity-time graphs	The distance on a velocity time graph is represented by the area under the graph.		U611
Non-linear graphs	Estimating areas under non-linear graphs	To estimate the area under a curved graph, divide it into equal width trapeziums.		U882

Non-linear graphs	Equations of circles	The equation of a circle comes from Pythagoras' theorem, with the radius being the hypotenuse.		U567
Non-linear graphs	Equations of tangents	The gradient of the tangent is the negative reciprocal of the gradient of the radius.		U758