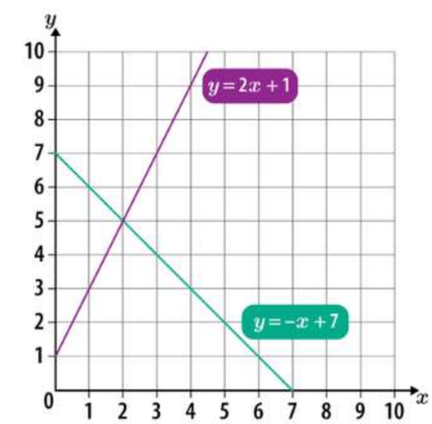
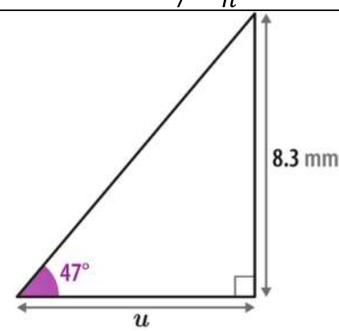
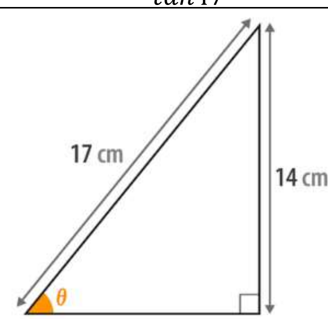
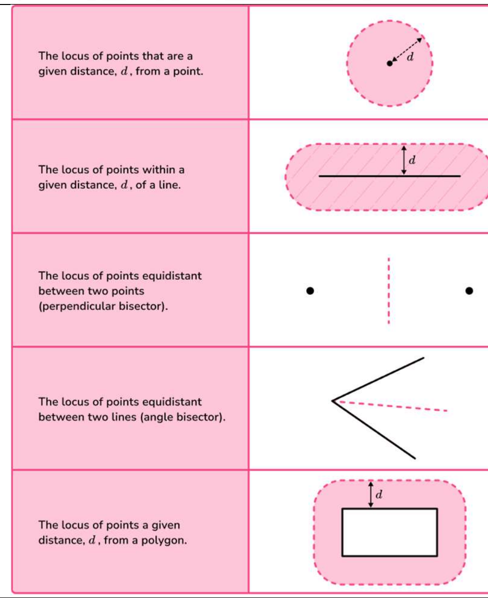
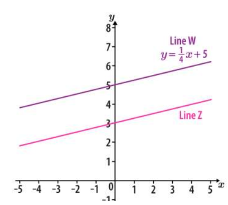
Maths Knowledge Organiser

Year: 10 Term: Autumn (Terms 1 and 2)

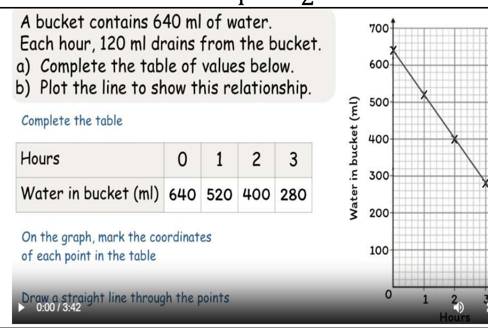
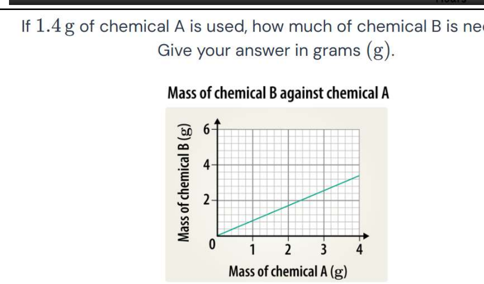
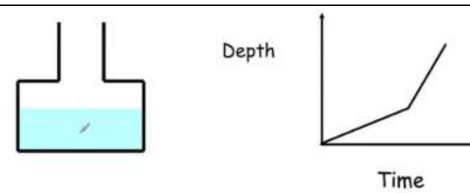
Unit	Topic/Skill	Definition/Tips	Example	Sparx Code
Repeated percentage change	Compound interest	Compound interest is interest calculated on the initial principal of a deposit or loan, as well as the accumulated interest from previous periods.	**Compound Interest** Where P is the principal amount, r is the intere rate over a given period and n is number of tin that the interest is compounded: $\text{Total accrued} = P\left(1+\frac{r}{100}\right)^n$ £1200 is invested in a bank account a rate of 2% for 5 years. How much is in the account at the end of the 5 years? $1200\left(1+\frac{2}{100}\right)^5 = £1324.90$	U671
	Repeated growth/decay	Growth and decay, in a mathematical context, refer to the direction in which a quantity is changing over time. Growth signifies an increase in value, while decay signifies a decrease.	Water in a tank is leaking at a rate of 5.5% per second. The tank is filled with 6 litres of water. How much water is in the tank after 8 seconds? $6\left(1-\frac{5.5}{100}\right)^8 = 3.82\ litres$	U286
Surface area and volume	Surface area of pyramids, cones, frustums, spheres and composite shapes	Surface area is the sum of all of the faces of a 3D shape	Curved surface of a cone == πrl Surface area of a sphere=$4\pi r^2$	U871 U523 U893 U334 U561
	Volume of pyramids, cones, frustums, spheres and composite shapes	Volume is the amount of space inside a 3D shape	Prism $V = area\ of\ cross\ section \times length$ Pyramid and cone $V = \frac{1}{3} \times base \times height$ Sphere $V = \frac{4}{3}\pi r^3$	U484 U116 U617 U350 U543

Linear simultaneous equations	Elimination	This means that we make the coefficients of either x or y the same and then add or subtract the equations	$6x+7y=27$ $8x+11y=41$ $48x+56y=216$ $48x+66y=246$ So $10y=30$ and y=3 Substituting back into the original equation: 6x+ 21=27 6x=6 and x=1	U760
	Substitution	This involves rearranging one equation and substituting into the other to find the missing variables.	$y=5x-4$ $y=3x+12$ Which gives 5x-4=3x+12 2x-4=12 2x=16 x=8 y = 5 x 8 -4 =36	U757
	Graphically	We look for the point of intersection for the solution.	 x=2 and y=5	U836
	Constructing and solving	These are questions in context for which we have to set up equations for.	The cost of 7 potted plants and 2 bags of fertiliser is £ The cost of 5 potted plants and 4 bags of fertiliser is £ Louis wants to buy 2 potted plants and 5 bags of fertili How much will this cost Louis? Let p be plants and b be bags. 7p+2b=43 5p+4b=41	U137

			14p+4b=86 5p+4b=41 9p=45 p=5 So a plant is £5 and a bag is £4	
Rearranging formulae	Two or more steps	We must do the same inverse operation to both sides of the equation.	Rearrange $cq-u=h$ to make q the subje $cq=h+u$ $q=\frac{h+u}{c}$	U181
	Where the subject appears more than once		Make x the subject of $c=7x-hx$ $c=x(7-h)$ $\frac{c}{7-h}=x$	U191
Right-angled trigonometry	Missing sides	Use SOHCAHTOA to determine which trigonometric ratio to use $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$	 $\tan 47 = \frac{8.3}{u}$ $u=\frac{8.3}{tan47}=7.74mm$	U283
	Missing angles		 $sin\theta = \frac{14}{17}$ $\theta = \sin^{-1}\frac{14}{17} = 55.4°$	U545

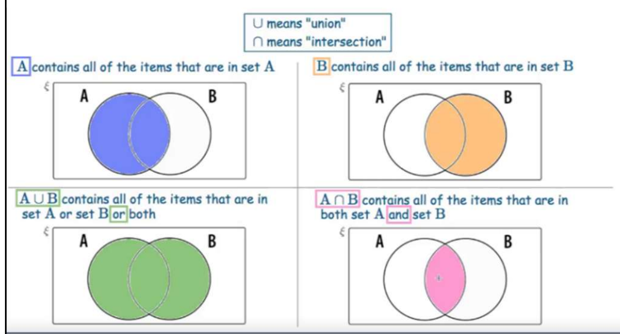
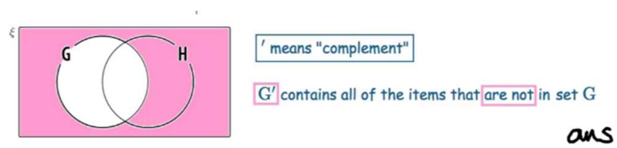
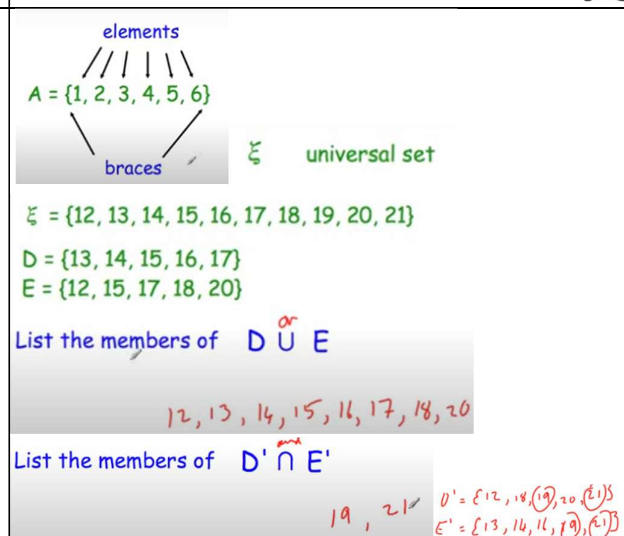
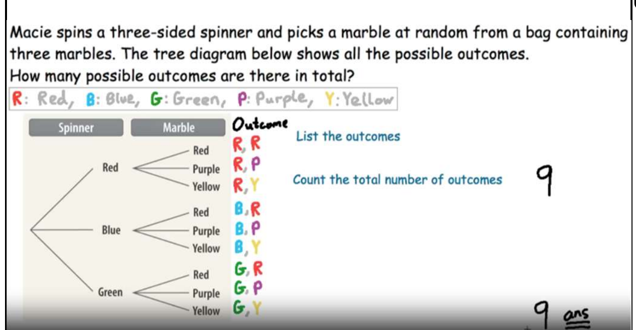
<table>
<tr><td></td><td>Exact values</td><td>These are most likely to appear on the non-calculator papers and are to be learned.</td><td>
<table>
<tr><th></th><th>0°</th><th>30°</th><th>45°</th><th>60°</th><th>90°</th></tr>
<tr><td>sin</td><td>0</td><td>$\frac{1}{2}$</td><td>$\frac{\sqrt{2}}{2}$</td><td>$\frac{\sqrt{3}}{2}$</td><td>1</td></tr>
<tr><td>cos</td><td>1</td><td>$\frac{\sqrt{3}}{2}$</td><td>$\frac{\sqrt{2}}{2}$</td><td>$\frac{1}{2}$</td><td>0</td></tr>
<tr><td>tan</td><td>0</td><td>$\frac{\sqrt{3}}{3}$</td><td>1</td><td>$\sqrt{3}$</td><td>X</td></tr>
</table>
</td><td>U627</td></tr>
<tr><td>Constructions</td><td>Constructing loci</td><td>"Loci" is the plural form of the Latin word "locus," which generally refers to a location or position. In mathematics, loci (plural) are sets of points that satisfy specific geometric conditions</td><td>
</td><td>U820</td></tr>
<tr><td>Linear graphs</td><td>Equations of parallel lines</td><td>Parallel lines have the same gradients.</td><td>Line W and line Z are parallel.
Give the equation of line Z in the form $y = mx + c$ where m and c are integers or fractions in their simplest


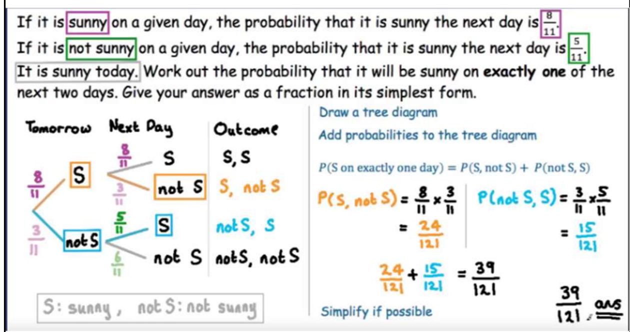
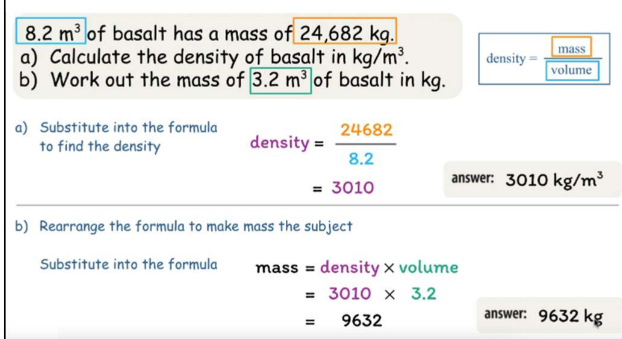
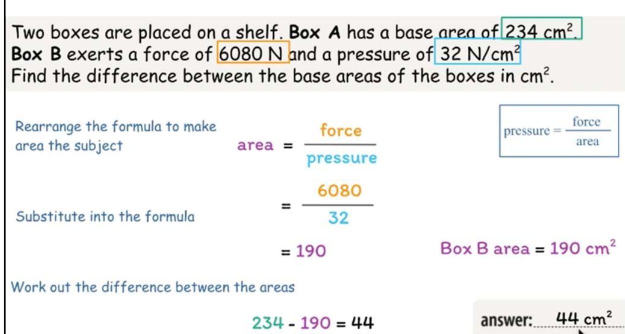
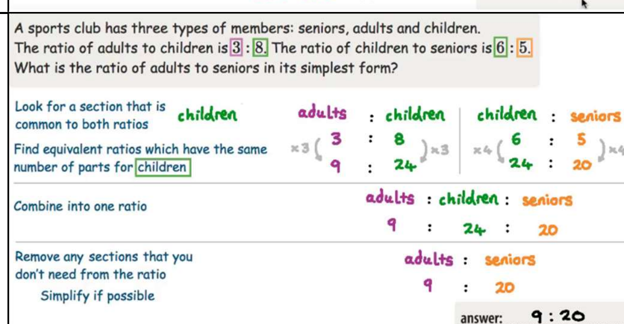
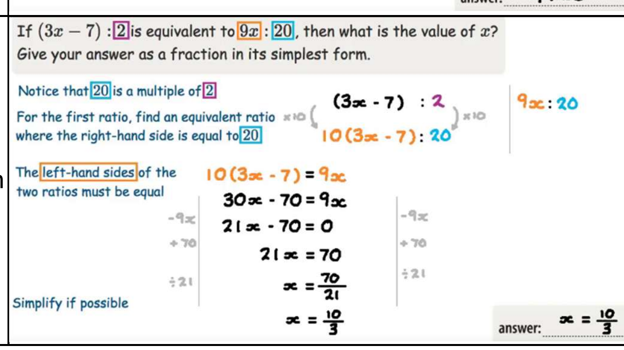
$$y = \frac{1}{4}x + 3$$</td><td>U377</td></tr>
<tr><td></td><td>Equations from a gradient and point</td><td>Use y=mx+c with the gradient and point to find c</td><td>A line has gradient 4 and passes through (2,11). Find the equation of the line.
$$11 = 2 \times 4 + c$$
$$c = 3$$
$y = 4x + 3$ is the equation.</td><td>U477</td></tr>
<tr><td></td><td>Equation from two points</td><td>We need to find the gradient using $\frac{chang\ \ in\ y}{change\ in\ x}$ and then use this with y=mx+c to find the c value.</td><td>Work out the equation of the line which passes through (6,8) and (9,23)
$$m = \frac{23 - 8}{9 - 6} = 5$$
$$8 = 5 \times 6 + c$$
$$c = -22$$</td><td>U848</td></tr>
</table>

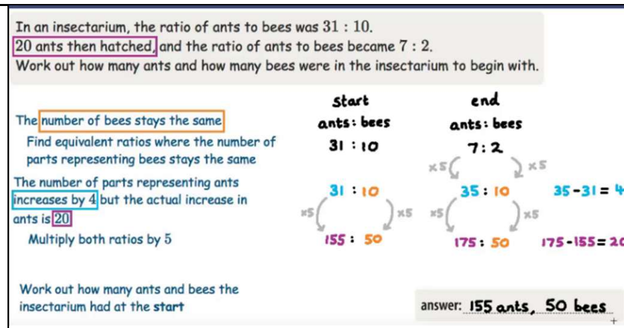
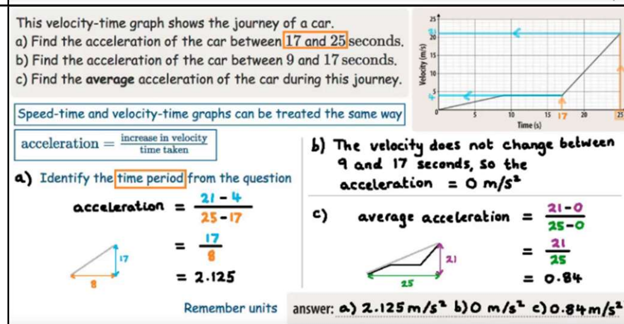
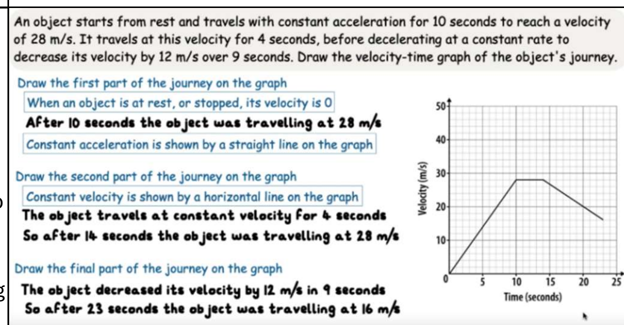
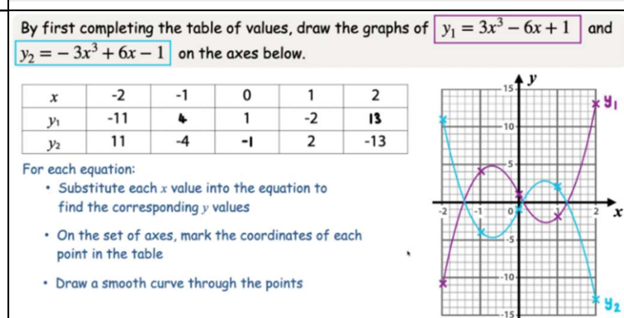
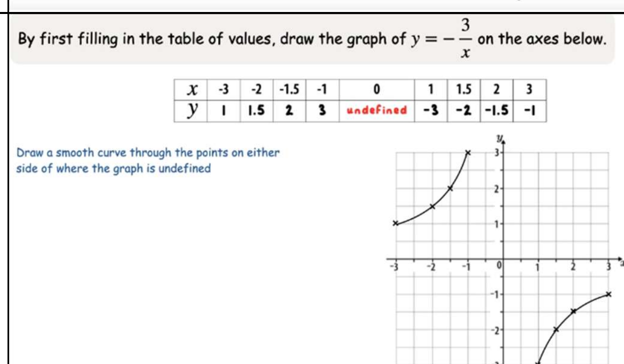
			$y = 5x - 22$	
	Equations of perpendicular lines	Perpendicular lines have negative reciprocal gradients. $m2 = -\frac{1}{m1}$	A line passes through (2,3) and is perpendicular to y=4x+1. What is the equation of the line? $m = -\frac{1}{4}$ $3 = -\frac{1}{2} + c$ $c = \frac{7}{2}$ $y = \frac{1}{4}x + \frac{7}{2}$	U898
Real life graphs	Plotting real life graphs	Plot the coordinates first and then join them up.		U652
	Interpreting real life graphs	Use a ruler to help you take the most accurate reading.	 1.2g	U862
	Water flows	These graphs show the rate of change of the depth of the water based on the shape of the container.		U896

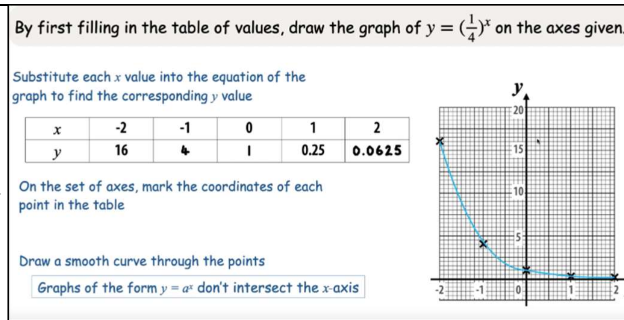
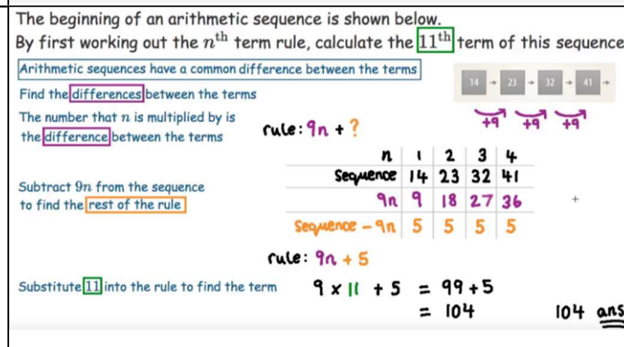
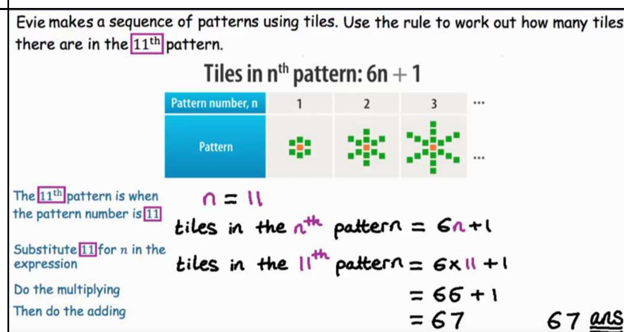
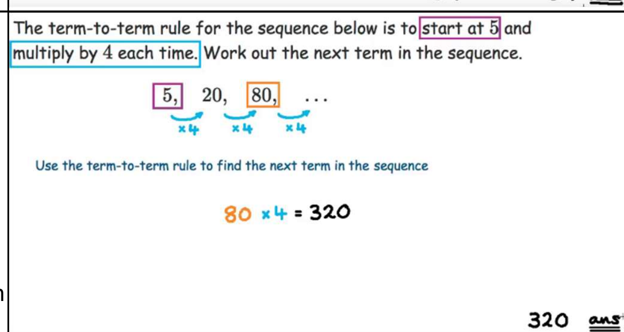
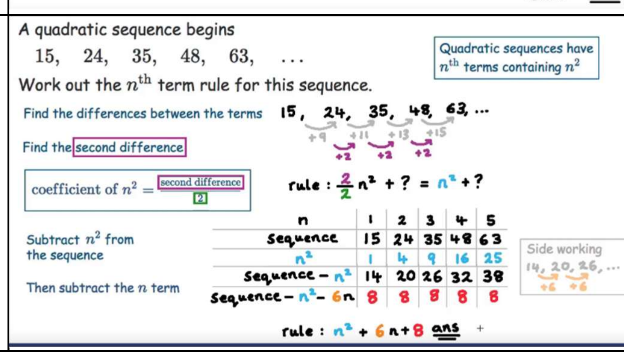
Maths Knowledge Organiser

Year: 10 Term: Spring (term 3 and 4)

Unit	Topic/Skill	Definition/Tips	Example	Sparx Code
Set Notation	Venn diagrams with set notation	Set Union Intersection Complement	  	U748
	Using set notation	Braces Element Universal Set	 	U296
Tree Diagrams	Tree diagrams for independent events	A tree diagram lists all possible outcomes for two or more events. Independent events do not affect each other. Event Outcomes Independent	 	U558

	Tree diagrams for dependent events	The events are dependent if the probability of the second event happening depends on the first. Multiply probabilities along the branches and add to combine at the end.		U729
Compound Measures	Calculating with density	**Density** is a way of comparing how heavy different materials are. $density = \frac{mass}{volume}$ The standard units of density are kg/m³ and g/cm³.		U910
	Calculating with pressure	The pressure is the force exerted per unit area. $pressure = \frac{force}{area}$ The standard units of pressure are N/m^2		U527
Working with ratio	Combining ratios	Multiplying ratios Equivalent ratio Lowest common multiple Simplify		U921
	Calculating with ratios and algebra	Scale up one or both ratios until one element is equal. Form an equation and solve to find x.		U676

	Changing ratios	Identify what stays the same after the change (e.g. number of bees) Then use equivalent ratios.		U865
Velocity-time graphs	Calculating acceleration from velocity-time graphs	Acceleration is the rate of change of velocity. It is the gradient of a velocity time graph. The units of acceleration are usually m/s^2		U562
	Plotting velocity-time graphs	A positive gradient indicates acceleration. A negative gradient indicates deceleration. A horizontal line (zero gradient) indicates the object is stationary or traveling at constant velocity.		U937
Cubic, reciprocal and exponential graphs	Graphs of cubic functions	Cubic graphs have up to 2 turning points.		U980
	Graphs of reciprocal functions	A reciprocal function is in the form $y = \frac{k}{x}$ Because the result of dividing a number by zero is undefined, the graph is discontinuous.		U593

<table>
<tr><td></td><td>Graphs of exponential functions*</td><td>An exponential function is in the form $y = ab^x$
When $b > 1$ the graph curves up steeply left to right.
When $0 < b < 1$ the graph curves down left to right.</td><td>
</td><td>U229</td></tr>
<tr><td>Sequences</td><td>Position-to-term rules for arithmetic sequences</td><td>A sequence is a list of numbers in a certain order.
Each number in the sequence is a term.
Arithmetic sequences have a common difference. They go up or down by the same amount each time.
The nth term helps you to find any term in the sequence</td><td>
</td><td>U498</td></tr>
<tr><td></td><td>Position-to-term rules for sequences of patterns</td><td>Terms in a sequence can also be patterns or shapes. Convert the pattern into a number sequence to find the nth term.</td><td>
</td><td>U978</td></tr>
<tr><td></td><td>Position-to-term rules for geometric sequences</td><td>In a geometric sequence the next term is found by multiplying the term before it by a fixed number. The number you multiply by is called the common ratio.</td><td>
</td><td>U958</td></tr>
<tr><td>Sequences (Higher Only)</td><td>Position-to-term rules for quadratic sequences</td><td>In a quadratic sequence the second difference is constant. If you halve the second difference you can find the coefficient of n^2 in the nth term.</td><td>
</td><td>U206</td></tr>
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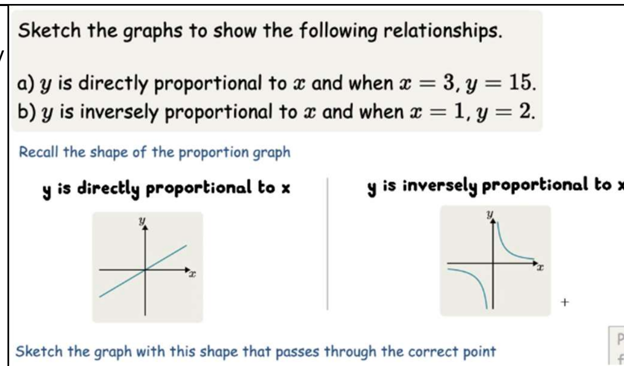
	Position-to-term rules for geometric sequences	The nth term for a geometric sequence is in the form $u_n = ar^{n-1}$ Where n is the position number, a is the first term and r is the common ratio.	The term-to-term rule for a sequence is to $\boxed{\text{start at 3}}$ and $\boxed{\text{multiply by 11 each time.}}$ Below are some statements that describe terms in the sequence. Work out the missing index in each of the statements. a) Term 2 is given by $3 \times 11^{\square}$ b) Term 6 is given by $3 \times 11^{\square}$ c) Term n is given by $3 \times 11^{\square}$ Use the term-to-term rule to work forwards through the sequence Term 1 $3 \xrightarrow{\times 11}$ Term 2 $33 \xrightarrow{\times 11}$ Term 3 $363 \xrightarrow{\times 11} \dots$ Write each term as 3 multiplied by a power of 11 Term 1: $3 = 3 \times 11^0$ Term 2: $3 \times \underbrace{11}_{1\ \text{copy}} = 3 \times 11^1$ Term 3: $3 \times \underbrace{11 \times 11}_{2\ \text{copies}} = 3 \times 11^2$ Term n: $3 \times \underbrace{11 \times 11 \dots \times 11}_{(n-1)\ \text{copies}} = 3 \times 11^{(n-1)}$ The index is equal to the number of copies we are multiplying together a) 1 , b) 5 , c) $(n-1)$ ans	U958
	Special sequences	The special sequences that you need to recognise are: even, odd, squares, cubes and triangular numbers. We also see Fibonacci type sequences.	Name | Sequence | n^{th} term rule Position: 1, 2, 3, 4, 5, … | n Even numbers: 2, 4, 6, 8, 10, … | $2n$ Odd numbers: 1, 3, 5, 7, 9, … (+2, +2, +2, +2) | $2n-1$ Square numbers: 1, 4, 9, 16, 25, … | n^2 Cube numbers: 1, 8, 27, 64, 125, … | n^3 Triangular numbers: 1, 3, 6, 10, 15, … (+2, +3, +4, +5) | $\frac{n(n+1)}{2}$ In a Fibonacci-type sequence, after the first two terms, each term is found by $\boxed{\text{adding together the two terms before it}}$ a) Continue the sequence for another 4 terms 0, 1, 1, 2, 3, 5, … 0+1, 1+1, 1+2, 2+3 answer: a) 1, 2, 3, 5	U680
	Combinations of sequences	In sequences of fractions, look for separate sequences in the numerator and denominator.	The start of a pattern of numbers is shown below. Write down an expression for the n^{th} term in this pattern. $\frac{1}{2} \to \frac{1}{5} \to \frac{1}{10} \to \frac{1}{17} \to \frac{1}{26} \to$ Only the denominators of the fractions are changing Work out the pattern in the denominators $2 \to 5 \to 10 \to 17 \to 26 \to$ Notice that the denominators are 1 more than the square numbers $1 \to 4 \to 9 \to 16 \to 25 \to$ n^{th} denominator : n^2+1 n^{th} term : $\frac{1}{n^2+1}$ $\frac{1}{n^2+1}$ ans	
Proportion	Interpreting direct proportion equations	If two sets of values are in direct proportion then one value is always a fixed number of times bigger than the other. The number you multiply by is called the constant of proportionality, k.	The number of minutes, n, that it takes to paint a wall is directly proportional to the area of the wall, a m^2. The equation of proportionality is $n = 9a$. Work out the area of a wall that takes $\boxed{63\ \text{minutes}}$ to paint. Substitute in the given value for n $n = 9a$ Solve to find a $63 = 9a$ ÷9 $7 = a$ Remember units answer: $7\ m^2$	U640
	Interpreting inverse proportion equations	If two sets of values are in inverse proportion then the two values always multiply together to give the same number. This number is called the constant of proportionality, k. $yx = k$ So $y = \frac{k}{x}$	$y = \frac{k}{x}$ If $\boxed{y=4}$ when $\boxed{x=12}$, what is the value of $\boxed{k}$? Work out the value of k $y = \frac{K}{x}$ Make the substitutions $4 = \frac{K}{12}$ Multiply each side by 12 ×12 $48 = K$ 48 ans	U364

<table>
<tr>
<td></td>
<td>Graphs of direct and inverse proportion</td>
<td>Direct and indirect proportion are easy to spot from their graphs.</td>
<td>
</td>
<td>U238</td>
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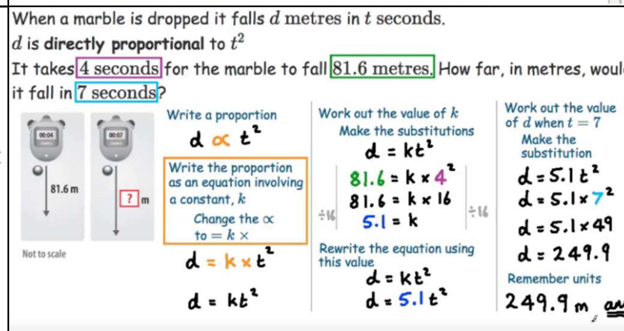
<tr>
<td>Proportion (Higher only)</td>
<td>Constructing direct proportion equations</td>
<td>Use the information in the question to write a direct proportion equation and use it to find any missing values.
Direct proportion equations are in the form $y = kx$</td>
<td>
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<td>U407</td>
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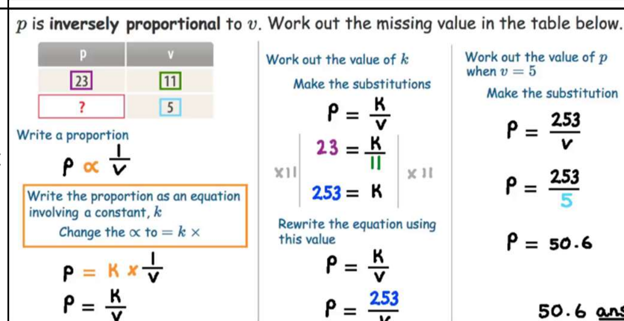
<tr>
<td></td>
<td>Constructing inverse proportion equations</td>
<td>Use the information in the question to write a direct proportion equation and use it to find any missing values. Inverse proportion equations are in the form $y = \frac{k}{x}$</td>
<td>
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<td>U138</td>
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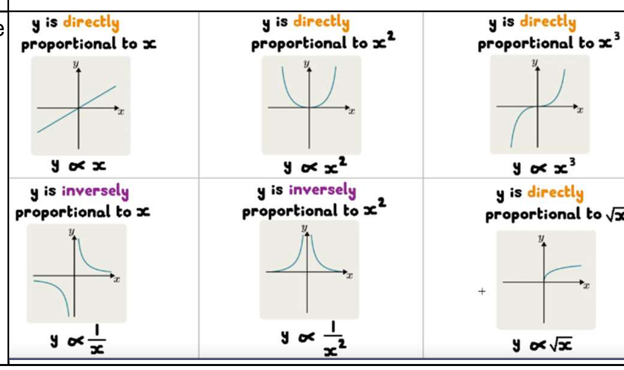
<tr>
<td></td>
<td>Graphs of direct and inverse proportion</td>
<td>It is important to be able to recognise and use the graphs of direct and inverse proportion that involve squares, cubes, square roots and cube roots.</td>
<td>
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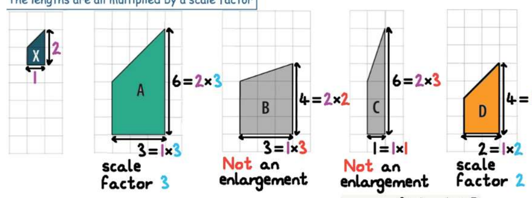
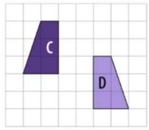
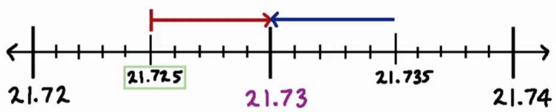
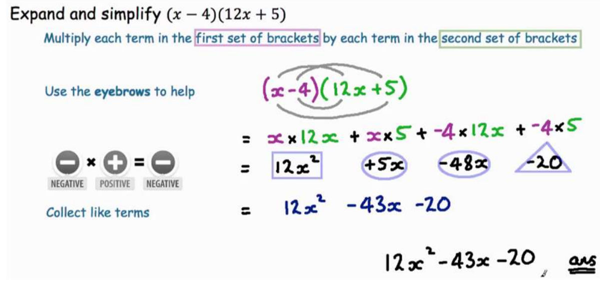
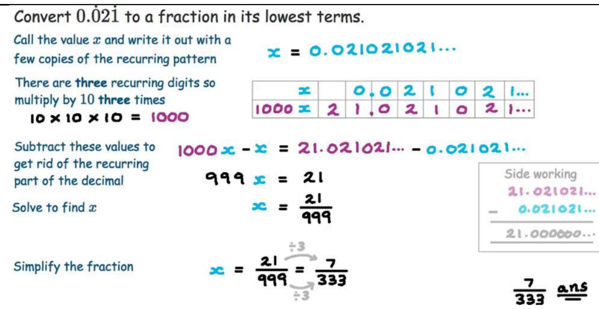
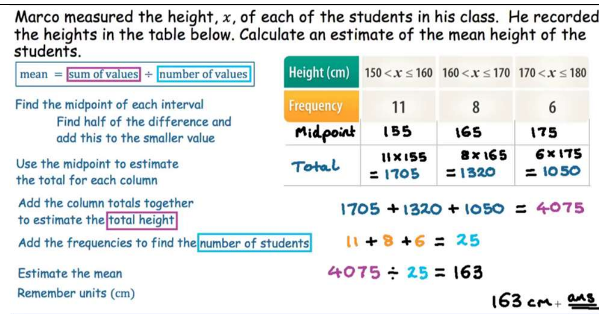
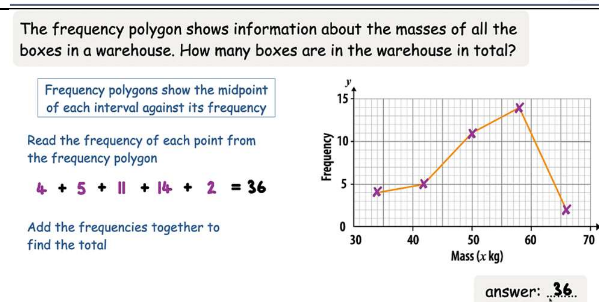
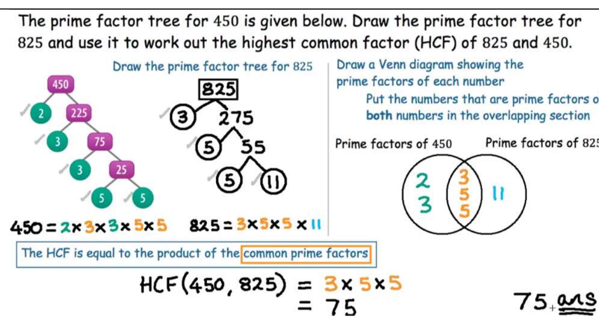
Maths Knowledge Organiser

Year: 10 Term: Summer (term 5 and 6)

Unit	Topic/Skill	Definition/Tips	Example	Sparx Code
Transformations	Enlarging by a positive and negative scale factor	Enlargement changes the size of a shape. All of the sides are multiplied by a scale factor	a) Select all of the shapes which are enlargements of shape X. b) Which shape shows an enlargement of shape X by a scale factor of 3? Enlarging a shape changes its size The lengths are all multiplied by a scale factor  answer: a) A and D	U134
	Combining Transformations	Multiple transformations can be performed on a shape	What has changed? Identify the type of transformation that does this Sloping side swapped from left to right side of shape — Reflection Shape moved down — Translation  answer: Reflection, Translation	U766
Rounding	Error Intervals	Identifying the smallest and/or biggest number that would round to a given number	I think of a number that has 3 decimal places. My number rounds to 21.73 to 2 decimal places. What is the smallest number that I could be thinking of? Use a number line  Mark the values that round to 21.73 to 2 d.p. What is the smallest value? 21.725 ans	U657
	Bounds	Finding the error interval of the result of a calculation using rounded numbers	A number, x, rounded to 2 d.p. is 25.34 Another number, y, rounded to 2 d.p. is 17.68 What are the lower and upper bounds of $x - y$? Work out the lower bound of x: 25.34 − 0.005 = 25.335 Work out the upper bound of x: 25.34 + 0.005 = 25.345 Work out the lower bound of y: 17.68 − 0.005 = 17.675 Work out the upper bound of y: 17.68 + 0.005 = 17.685 Work out the lower bound of x - y: Subtract the upper bound of y from the lower bound of x lower bound = 25.335 − 17.685 = 7.65 Work out the upper bound of x - y: Subtract the lower bound of y from the upper bound of x upper bound = 25.345 − 17.675 = 7.67 answer: lower bound = 7.65, upper bound = 7.67	U587
Index Laws	Simplifying expressions using index laws	Index laws allow you to combine and simplify expressions involving indices	Fully simplify $(3k^{12})^3$ $\boxed{(x \times y)^a = x^a \times y^a}$ $\boxed{(x^a)^b = x^{(a \times b)}}$ $(3k^{12})^3$ $= (3 \times k^{12})^3$ $= (3)^3 \times (k^{12})^3$ $= 27 \times k^{(12 \times 3)}$ $= 27 \times k^{36}$ $= 27k^{36}$ $27k^{36}$ ans	U662

Brackets	Expanding and factorising brackets	When expanding double brackets every term in the first bracket must be multiplied by every term in the second		U768, U178
Recurring Decimals (H)	Fractions and recurring decimals	All recurring decimals can be written as fractions		U550, U689
Handling Data and Statistical Diagrams	Grouped data and averages	A frequency table can be used to help summarize large sets of data.		U877
	Drawing and interpreting statistical diagrams	Diagrams can provide a better visual representation of data		U200, U590, U840, U642, U879
Factors, Multiples and Primes (F)	HCF and LCM	The highest common factor (HCF) is the biggest number that goes into a set of numbers. The lowest common multiple (LCM) is the smallest number that a set of numbers go into		U751, U529, U250

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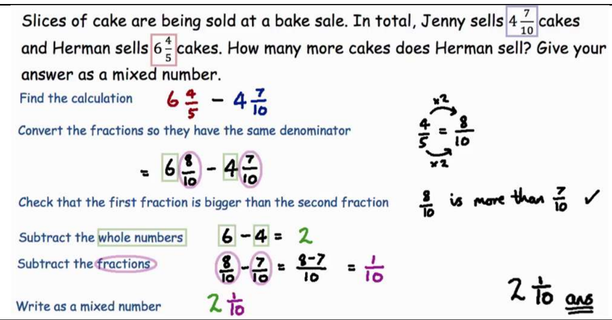
<tr><td>Fractions (F)</td><td>Fractions and mixed numbers</td><td>Factions greater than 1 can be represented as mixed numbers</td><td>
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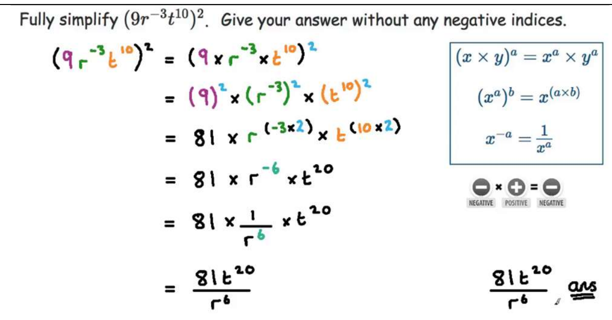
<tr><td>Expressions (F)</td><td>Simplifying expressions</td><td>Algebraic expressions can be simplified by collecting like terms, using the index laws and by cancelling common factors</td><td>
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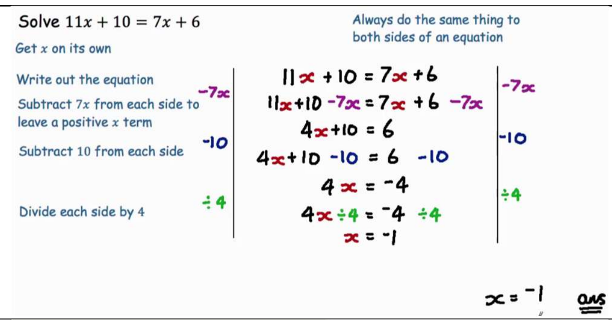
<tr><td>Equations (F)</td><td>Solving Equations</td><td>Solving an equation means finding the value(s) of an unknown variable that make the equation true</td><td>
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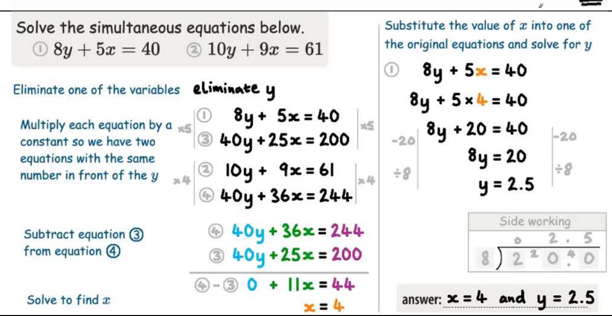
<tr><td></td><td>Simultaneous Equations</td><td>If there is more than 1 unknown, then you will need to solve more than 1 equations simultaneously</td><td>
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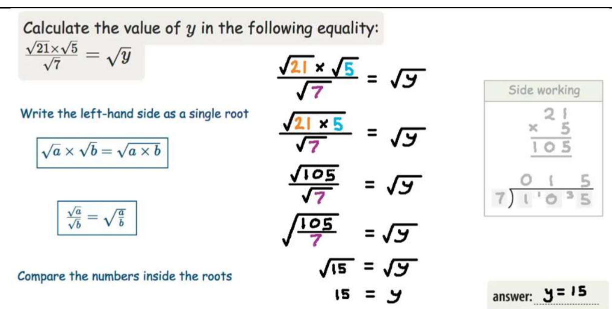
<tr><td>Surds (H)</td><td>Multiplying and dividing surds</td><td>Surds can be multiplied and divided by multiplying or dividing the numbers inside the root.</td><td>
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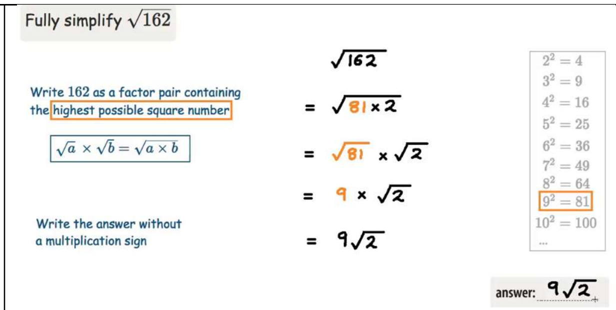
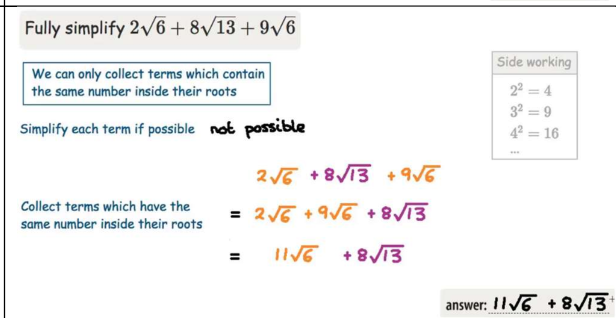
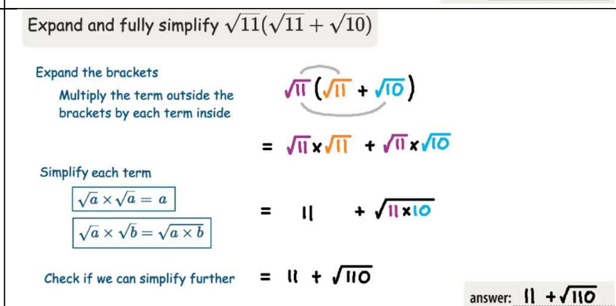
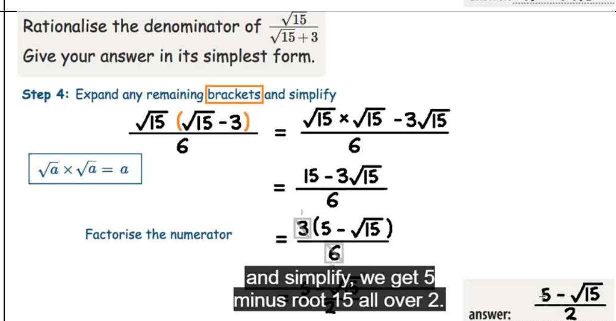
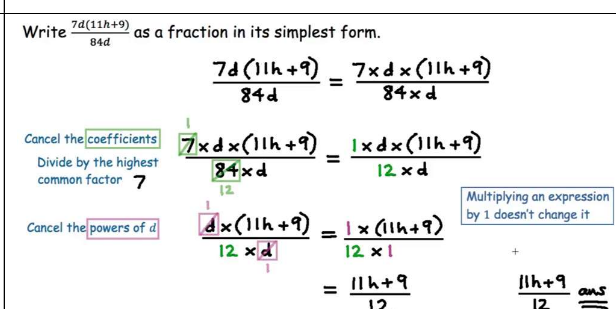
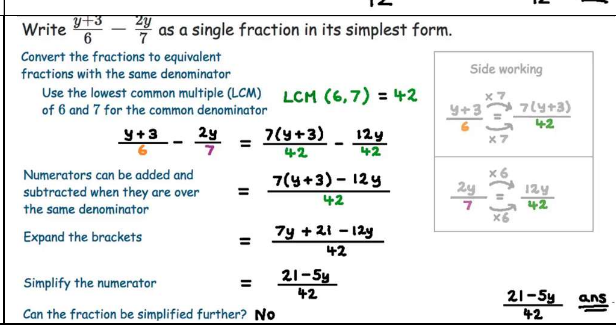
	Simplifying surds	Surds can be simplified if the number in the surd has a square number as a factor		U338
	Adding and subtracting surds	Surds with the same numbers under the roots can be added or subtracted		U872
	Expanding brackets with surds	Expressions with brackets including surds can be multiplied out or expanded		U499
	Rationalising denominators	A fraction with a surd in the denominator can be simplified so that the denominator is an integer		U707, U281
Algebraic fractions	Simplifying algebraic fractions by factorising	Factorising the numerator and denominator will help to identify any common factors		U437, U294
	Adding and subtracting algebraic fractions	Just like with ordinary fractions, you need a common denominator when adding or subtracting		

		algebraic fractions		
	Multiplying algebraic fractions	Multiply the numerators and denominators and simplify if necessary	Write the expression below as a fraction in its simplest form. $\frac{7f}{10(f-11)} \times \frac{f-11}{4}$ $\frac{7f}{10(f-11)} \times \frac{f-11}{4} = \frac{7f \times (f-11)}{10(f-11) \times 4}$ Cancel any $\boxed{\text{common factors}}$ that appear in both the numerator and denominator $= \frac{7f \times \boxed{(f-11)}}{10\boxed{(f-11)} \times 4}$ $= \frac{7f}{10 \times 4}$ Check the answer is fully simplified ✓ $= \frac{7f}{40}$ $\frac{7f}{40}$ ans	U178
	Dividing algebraic fractions	When dividing fractions, multiply by the reciprocal, i.e. multiply by the second fraction ‘flipped’	Work out $\frac{7y^{13}}{z^9} \div \frac{12}{y^{11}z^{20}}$. Give your answer as a fraction in its simplest form. Write the division as a multiplication Change the ÷ to × Flip the second fraction $\frac{7y^{13}}{z^9} \div \frac{12}{y^{11}z^{20}} = \frac{7y^{13}}{z^9} \times \frac{y^{11}z^{20}}{12}$ Simplify the fraction if possible Cancel the $\boxed{\text{variables}}$ $x^a \div x^b = x^{(a-b)}$ $x^a \times x^b = x^{(a+b)}$ Side working $z^{20} \div z^9 = z^{(20-9)} = z^{11}$ $y^{13} \times y^{11} = y^{(13+11)} = y^{24}$ $= \frac{7 \times y^{13} \times y^{11} \times \cancel{z^{20}}^{z^{11}}}{\cancel{z^9}_{1} \times 12}$ $= \frac{7 \times y^{13} \times y^{11} \times z^{11}}{1 \times 12}$ $= \frac{7y^{24}z^{11}}{12}$ Check that the answer is fully simplified ✓ $\frac{7y^{24}z^{11}}{12}$ ans	U858